EQUIVARIANT METHODS IN REPRESENTATION THEORY

ALEXANDRE MINETS

Description: One of the major advances in representation theory of algebras in the late 20th century was the realization that for many algebras, their finite-dimensional modules can be found in cohomology groups of various topological spaces. Consequently, one can use geometric techniques to study purely algebraic question, and this line of thought has proved to be extremely fruitful. The topological spaces one considers often come equipped with an action of a big group of symmetries. One is therefore naturally led to look for a cohomology theory which remembers this action. Enter equivariant cohomology.

In this course, we will review the basic constructions of equivariant cohomology, with particular focus on the localization theorem, which will be our main computational tool. Having set up the foundations, we will proceed to study the geometry of various spaces, and see how their geometry reflects in the study of algebras of interest. Our main example will be flag varieties and Hecke algebras.

Prerequisites: basic algebraic topology (singular homology/cohomology), basic algebraic geometry (complex algebraic varieties). Some familiarity with representation theory would be helpful for the second half, but is not required.

Plan of the course:

(1) Definition of equivariant cohomology. Properties: functoriality, abelianization, Gysin maps. Computation in examples: Grassmannians, flag varieties;
(2) Localization theorem, integration formula. Equivariant formality, Białynicki-Birula decomposition. GKM description of equivariant cohomology;
(3) Cohomology of toric varieties, Stanley-Reisner ring. Schubert calculus on Grassmannians and flag varieties;
(4) NilHecke algebra and Demazure operators. Abelianization theorem with modular coefficients;
(5) Equivariant Borel-Moore homology. Intersection product. Convolution algebras and their localizations;
(6) Localization in equivariant K-theory. Comparison with cohomology via Chern character. Application: Weyl character formula;
(7) Springer resolution and affine Hecke algebras, character formulas for representations of Hecke algebras;
(8) Hilbert schemes of points, relation between torus fixed points and Macdonald polynomials;
(9) If time permits: Yangians and affine quantum groups as convolution algebras.

References

[1] Anderson D., Fulton W., *Equivariant cohomology in algebraic geometry*, 2023.
[2] Chriss N., Ginzburg V., *Representation theory and complex geometry*, 1997.
[3] Nakajima H., *Lectures on Hilbert schemes of points on surfaces*, 1999.

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